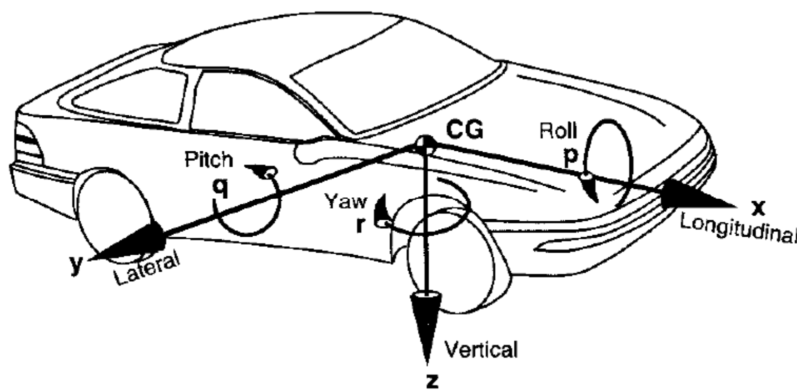
Suspension systems and components

Objectives

- To provide good ride and handling performance –
  - vertical compliance providing chassis isolation
  - ensuring that the wheels follow the road profile
  - very little tire load fluctuation
- To ensure that steering control is maintained during maneuvering –
  - wheels to be maintained in the proper position wrt road surface
- To ensure that the vehicle responds favorably to control forces produced by the tires during
  - longitudinal braking
  - accelerating forces,
  - lateral cornering forces and
  - braking and accelerating torques
  - this requires the suspension geometry to be designed to resist squat, dive and roll of the vehicle body
- To provide isolation from high frequency vibration from tire excitation
  - requires appropriate isolation in the suspension joints
  - Prevent transmission of 'road noise' to the vehicle body

Vehicle Axis system

- Un-sprung mass
- Right-hand orthogonal axis system fixed in a vehicle
- x-axis is substantially horizontal, points forward, and is in the longitudinal plane of symmetry.
- y-axis points to driver's right and
- z-axis points downward.
- Rotations:
  - A yaw rotation about z-axis.
  - A pitch rotation about y-axis.
  - A roll rotation about x-axis



SAE vehicle axes

Figure from Gillespie,1992

Tire Terminology - basic

- Camber angle
  - angle between the wheel plane and the vertical
  - taken to be positive when the wheel leans outwards from the vehicle
- Swivel pin (kingpin) inclination
  - angle between the swivel pin axis and the vertical
- Swivel pin (kingpin) offset
  - distance between the centre of the tire contact patch and
  - intersection of the swivel pin axis and the ground plane

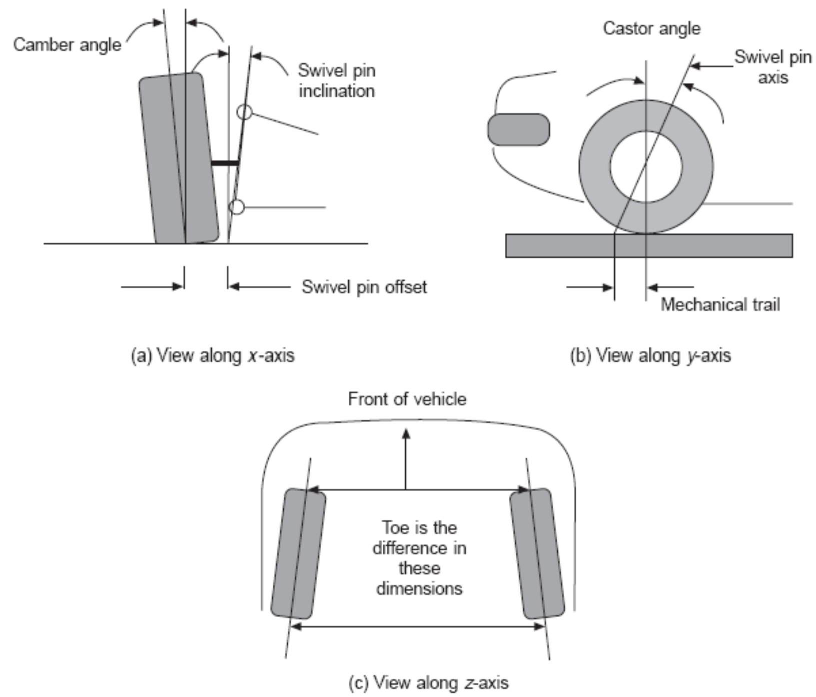


Figure from Smith,2002

Tire Terminology - basic

- Castor angle
  - inclination of the swivel pin axis projected into the fore–aft plane through the wheel centre
  - positive in the direction shown.
  - provides a self-aligning torque for non-driven wheels.
- Toe-in and Toe-out
  - difference between the front and rear distances separating the centre plane of a pair of wheels,
  - quoted at static ride height – toe-in is when the wheel centre planes converge towards the front of the vehicle

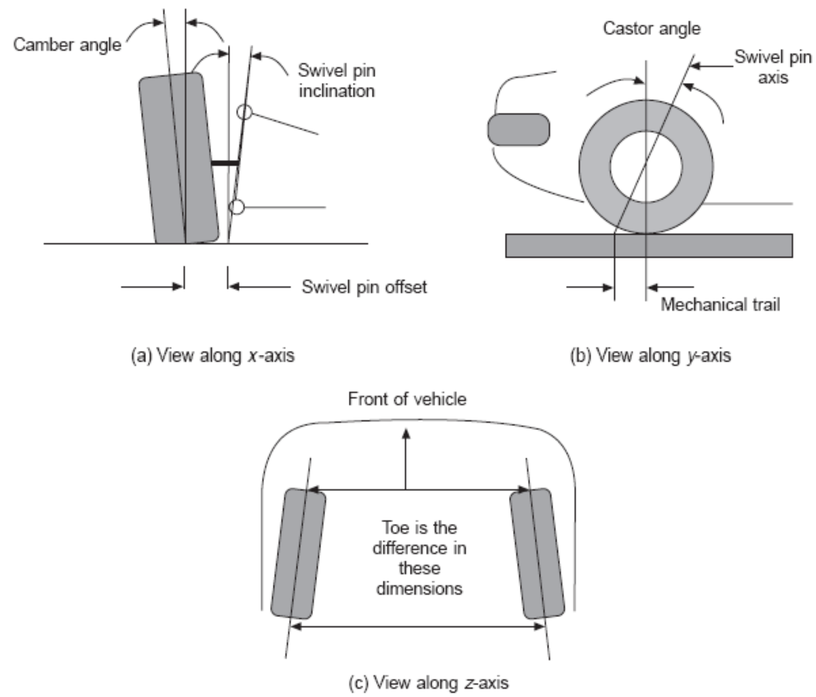


Figure from Smith,2002

The mobility of suspension mechanisms

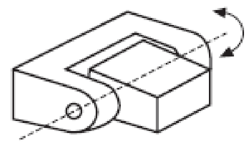
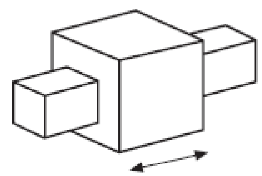
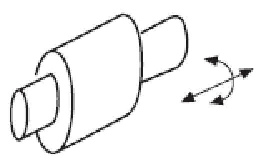
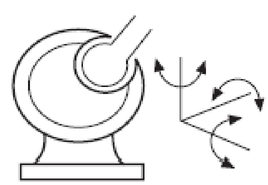
Name of pair	Geometric form	Degrees of freedom
Revolute R		1
Prism P		1
Cylinder C		2
Sphere S		3

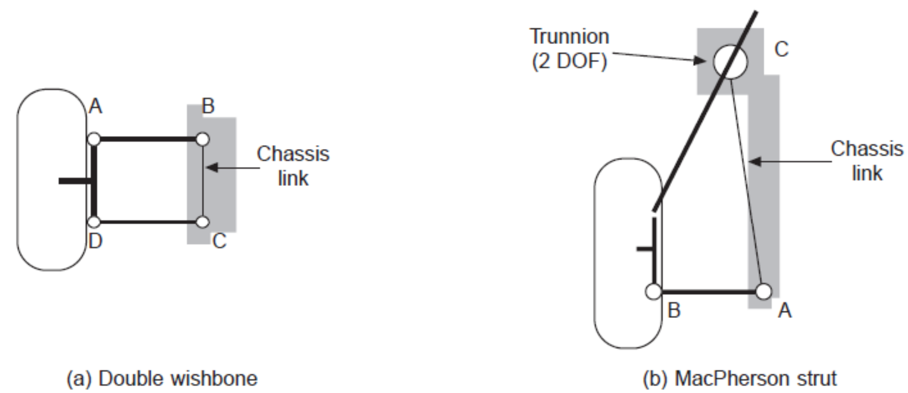
Figure from Smith,2002

Analysis of Suspension Mechanisms

- 3D mechanisms
- Compliant bushes create variable link lengths
- 2D approximations used for analysis
- Requirement
  - Guide the wheel along a vertical path
  - Without change in camber
- Suspension mechanism has various SDOF mechanisms

The mobility of suspension mechanisms

- Guide motion of each wheel along (unique) vertical path relative to the vehicle body without significant change in camber.
- Mobility (DOF) analysis is useful for checking for the appropriate number of degrees of freedom,
- Does not help in synthesis to provide the desired motion



Two-dimensional kinematics of common suspension mechanisms

$$M = 3(n - 1) - j_h - 2j_l$$

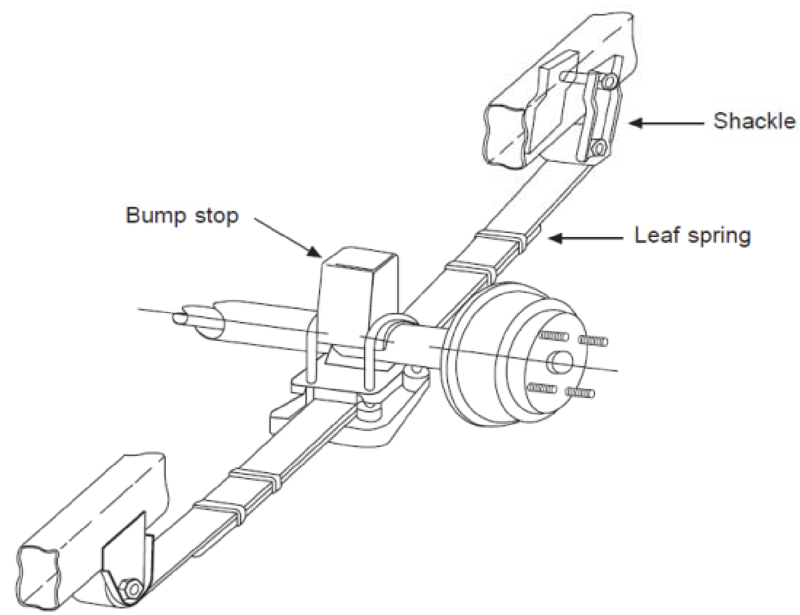
Figure from Smith,2002

Suspension Types -Dependent

- Motion of a wheel on one side of the vehicle is dependent on the motion of its partner on the other side
- Rarely used in modern passenger cars
  - Can not give good ride
  - Can not control high braking and accelerating torques
- Used in commercial and off-highway vehicles

Hotchkiss Drive

- Axle is mounted on longitudinal leaf springs, which are compliant vertically and stiff horizontally
- The springs are pin-connected to the chassis at one end and to a pivoted link at the other.
- This enables the change of length of the spring to be accommodated due to loading

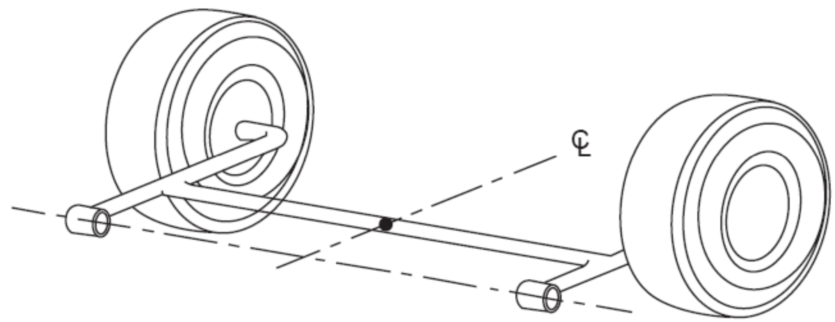


Hotchkiss Drive

Figure from Smith,2002

Semi-dependent Suspension

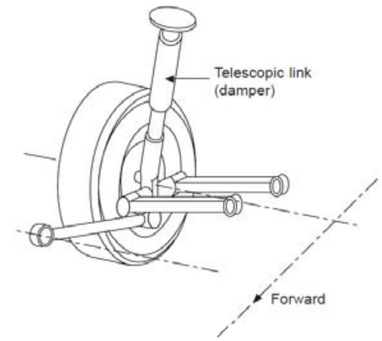
- the rigid connection between pairs of wheels is replaced by a compliant link.
- a beam which can bend and flex providing both positional control of the wheels as well as compliance.
- tend to be simple in construction but lack scope for design flexibility
- Additional compliance can be provided by rubber or hydro-elastic springs.
- Wheel camber is, in this case, the same as body roll



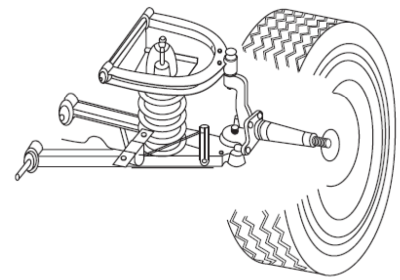
Trailing twist axle suspension

Suspension Types - Independent

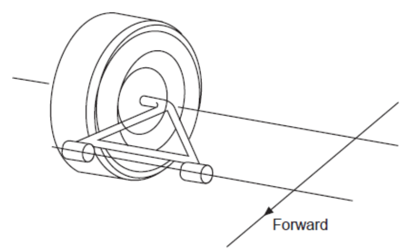
- motion of wheel pairs is independent, so that a disturbance at one wheel is not directly transmitted to its partner
- Better ride and handling



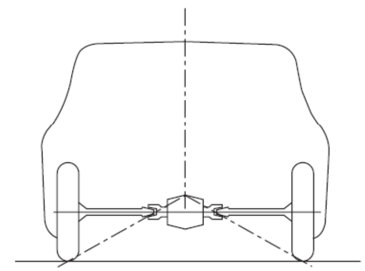
Macpherson Strut



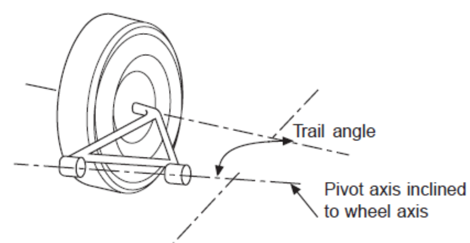
Double wishbone



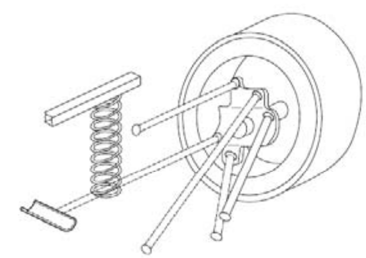
Trailing arm



Swing axle



Semi trailing arm

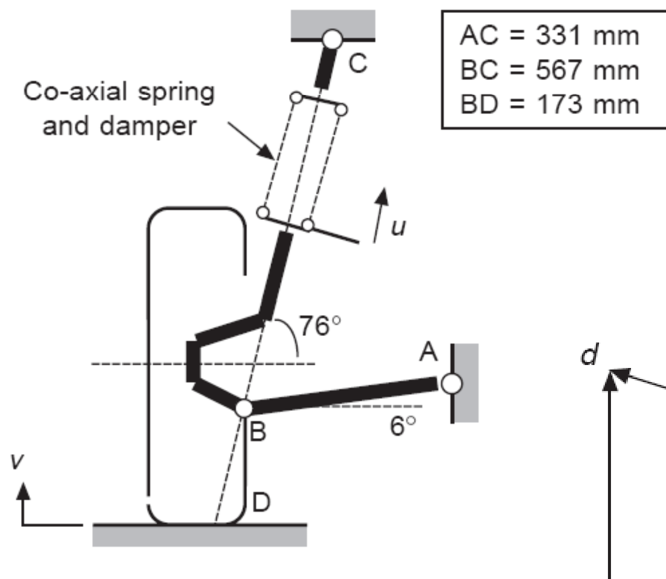


Multi-link

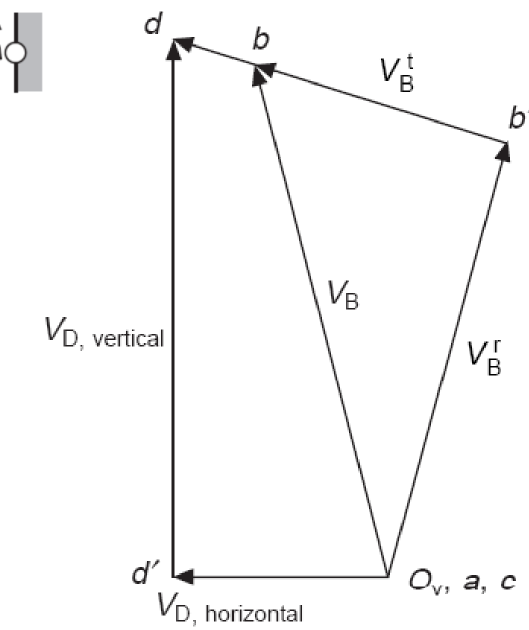
Kinematic Analysis -Graphical

Graphical Analysis

- Objective
  - The suspension ratio R (the rate of change of vertical movement at D as a function of spring compression)
  - The bump to scrub rate for the given position of the mechanism.



(a) Position of mechanism



(b) Velocity diagram

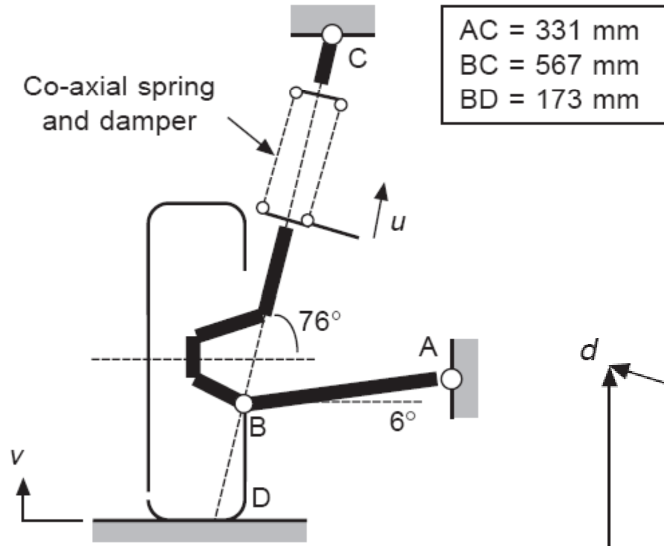
Figure from Smith,2002

Kinematic Analysis -Graphical

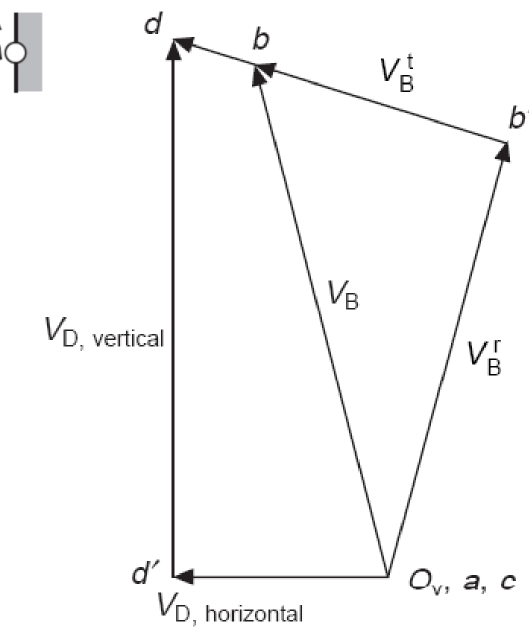
- Draw suspension mechanism to scale, assume chassis is fixed

$$V_B = \omega_{BA} r_{BA}$$

- Construct the velocity diagram



(a) Position of mechanism



(b) Velocity diagram

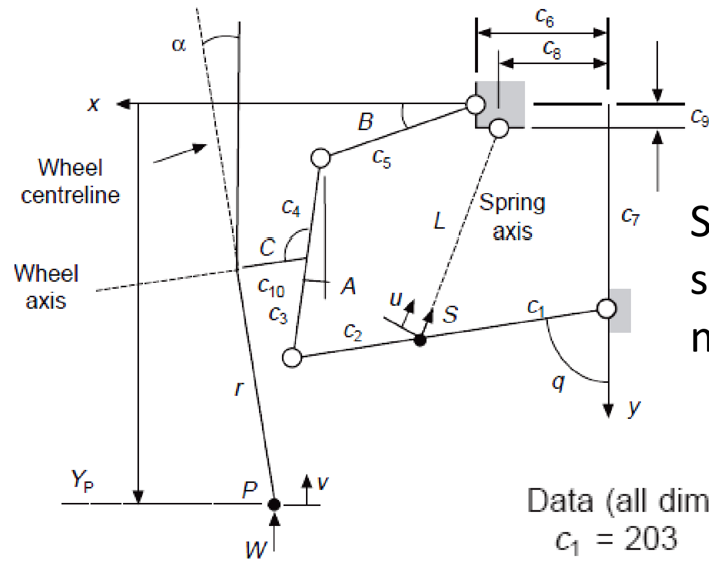
$$R = \frac{dv}{du} = \frac{dd'}{O_V b'} = \frac{311}{267} = 1.16$$

$$\text{Scrub to bump} = \frac{O_v d'}{dd'} = \frac{147.6}{311} = 0.47$$

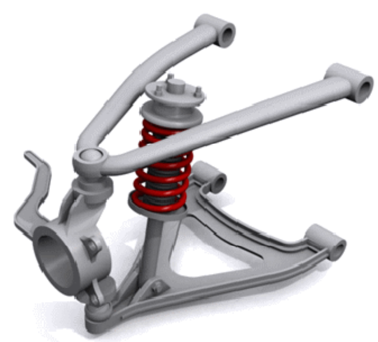
Figure from Smith,2002

Kinematic Analysis – Sample calculation

- Double wish bone
- The objectives are
  - Determine camber angle α, and suspension ratio R (as defined in the previous example)
  - For suspension movement described by "q" varying from 80° to 100°
  - Given that in the static laden position "q" = 90°.



Simplified suspension model



Data (all dimensions in mm):

c_1 = 203
c_2 = 102
c_3 = 127
c_4 = 127
c_5 = 152
c_6 = 127
c_7 = 267
c_8 = 102
c_9 = 51
c_{10} = 110
C = 96°
Tyre rolling radius r = 292

Figure from Smith,2002, Google search

Kinematic Analysis – Sample calculation

- Positions are provided
- Two non-linear equations solved for positions described interval 1°

Data: $c_1 := 203$ $c_2 := 102$ $c_3 := 127$ $c_4 := 127$ $c_5 := 152$ $c_6 := 127$

$c_7 := 267$ $c_8 := 102$ $c_9 := 51$ $c_{10} := 110$ $C := 96°$ $r := 292$

Constants $c_{12} := c_1 + c_2$ $c_{34} := c_3 + c_4$ $k_{dr} := \frac{\pi}{180}$

Solution estimates: $A := -10$ $B := 10$

Given

$$c_{12} \cdot \sin(q \cdot k_{dr}) - c_{34} \cdot \sin(A \cdot k_{dr}) - c_5 \cdot \cos(B \cdot k_{dr}) - c_6 = 0$$

$$c_{12} \cdot \cos(q \cdot k_{dr}) - c_{34} \cdot \cos(A \cdot k_{dr}) - c_5 \cdot \sin(B \cdot k_{dr}) + c_7 = 0$$

$$F(q) := \text{Find}(A, B)$$

$$q := 80..100 \quad i := 0..20$$

$$A_i := F(80 + i)_0 \quad B_i := F(80 + i)_1 \quad q_i := 80 + i$$

Kinematic Analysis

Camber angle (degrees) $\alpha_i := C - 90 - A_i$

Express angles in radians

$$q_{r_i} := q_i \cdot k_{dr} \quad A_{r_i} := A_i \cdot k_{dr} \quad B_{r_i} := B_i \cdot k_{dr} \quad \alpha_{r_i} := \alpha_i \cdot k_{dr}$$

Vertical position of tyre contact point

$$Y_{P_i} := c_7 + c_{12} \cdot \cos(q_{r_i}) - c_3 \cdot \cos(A_{r_i}) + c_{10} \cdot \sin(\alpha_{r_i}) + r \cdot \cos(\alpha_{r_i})$$

Mean position of tyre contact point: $Y_{PO} := Y_{P_{10}}$, $Y_{PO} = 432.644$ mm

Deflection from mean position: $v_i := Y_{P_i} - Y_{PO}$

Kinematic Analysis

The second part of the solution begins by expressing the length of the suspension spring in terms of the primary variable and then proceeds to determine the velocity coefficients

$K_{YP}(q) = \frac{dY_P}{dq}$ and $K_L(q) = \frac{dL}{dq}$. These allow the suspension ratio $R = \frac{K_{YP}}{K_L}$ to be determined.

Length of suspension spring

$$L_i := \sqrt{(c_1 \cdot \sin(q_{r_i}) - c_8)^2 + (c_7 + c_1 \cdot \cos(q_{r_i}) - c_9)^2}$$

Mean position of suspension spring $L_O := L_{10}$ $L_O = 238.447$ mm

Deflection from mean position $u_i := L_O - L_i$

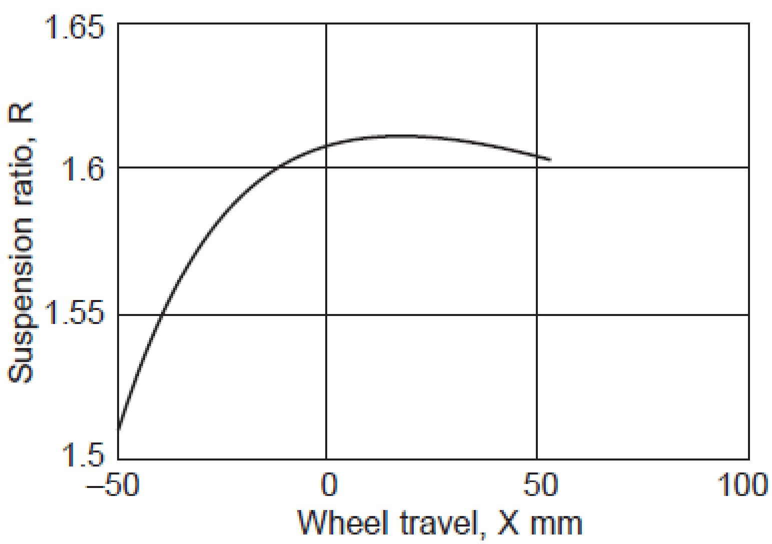
Velocity coefficients $K_{A_i} := \frac{c_{12} \cdot \cos(q_{r_i} + B_{r_i})}{c_{34} \cdot \cos(A_{r_i} + B_{r_i})}$

$$K_{YP_i} := c_{12} \cdot \sin(q_{r_i}) + K_{A_i} \cdot (c_3 \cdot \sin(A_{r_i}) - c_{10} \cdot \cos(\alpha_{r_i}) + r \cdot \sin(\alpha_{r_i})$$

$$K_{L_i} := \frac{c_1 \cdot c_9 \cdot \sin(q_{r_i}) - c_1 \cdot c_7 \cdot \sin(q_{r_i}) - c_1 \cdot c_8 \cdot \cos(q_{r_i})}{\sqrt{\left[(c_1 \cdot \sin(q_{r_i}) - c_8)^2 + (c_7 + c_1 \cdot \cos(q_{r_i}) - c_9)^2\right]}}$$

Suspension ratio $R_i := \frac{K_{YP_i}}{K_{L_i}}$ $R_{10} = 1.607$ **at static ride height**

Kinematic Analysis - Results



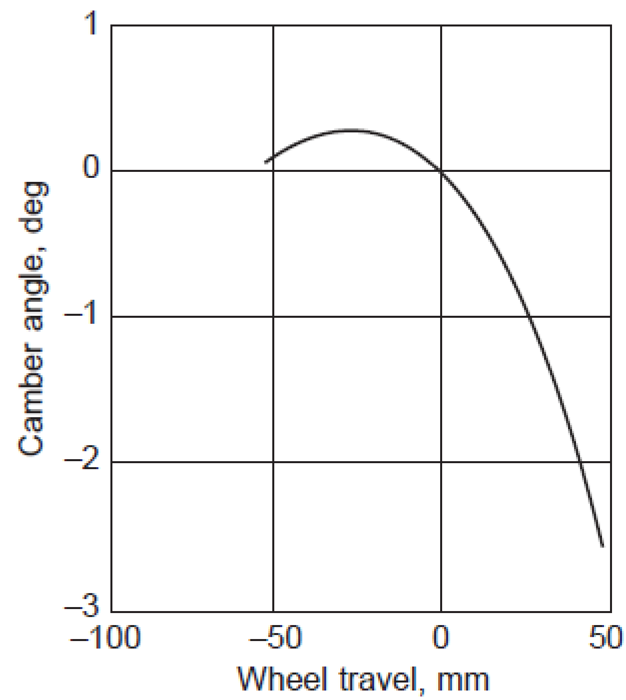


Figure from Smith,2002

Roll centre analysis

Two Definitions

- SAE : a point in the transverse plane through any pair of wheels at which a transverse force may be applied to the sprung mass without causing it to roll
- Kinematics : the roll centre is the point about which the body can roll without any lateral movement at either of the wheel contact areas

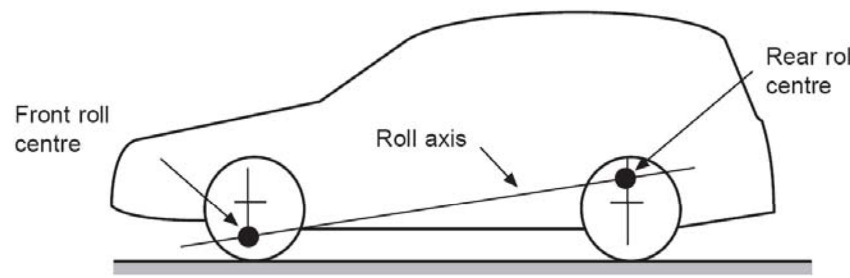


Figure 10.16 Roll axis location

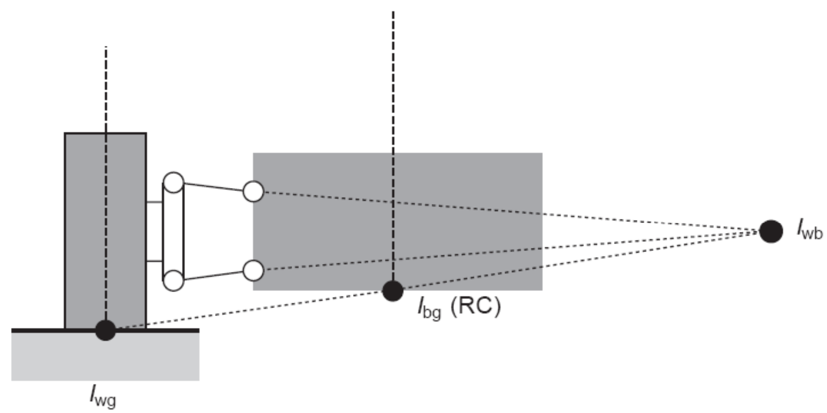
Figure from Smith,2002

Limitations of Roll Centre Analysis

- As roll of the sprung mass takes place, the suspension geometry changes, symmetry of the suspension across the vehicle is lost and the definition of roll centre becomes invalid.
  - It relates to the non-rolled vehicle condition and can therefore only be used for approximations involving small angles of roll
  - Assumes no change in vehicle track as a result of small angles of roll.

Roll-centre determination

- Aronhold–Kennedy theorem of three centers : when three bodies move relative to one another they have three instantaneous centers all of which lie on the same straight line
- I_{wb} can be varied by angling the upper and lower wishbones to different positions, thereby altering the load transfer between inner and outer wheels in a cornering maneuver.
- This gives the suspension designer some control over the handling capabilities of a vehicle

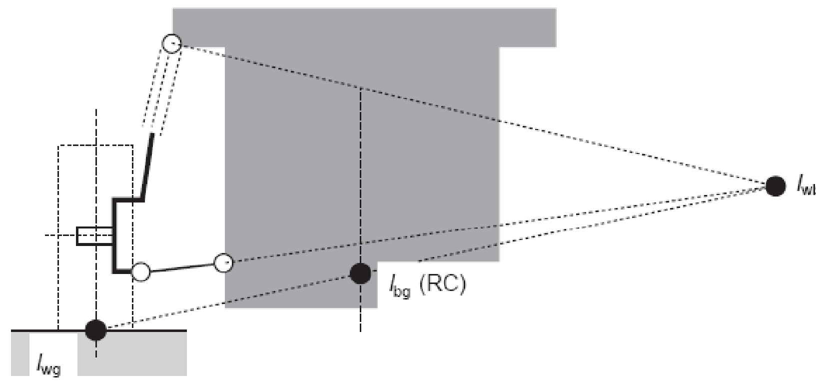


For a double wishbone

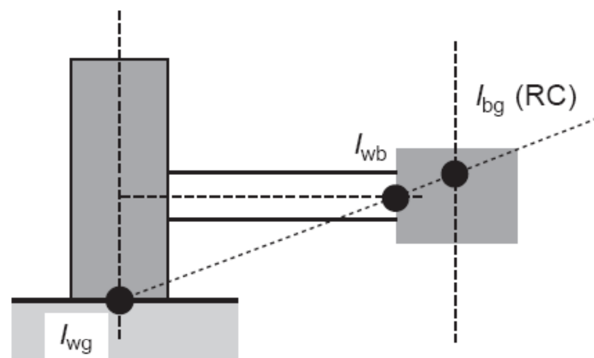
Figure from Smith,2002

Roll-centre determination

- In the case of the MacPherson strut suspension the upper line defining l_{wb} is perpendicular to the strut axis.
- Swing axle roll center is located above the "virtual" joint of the axle.



Macpherson strut

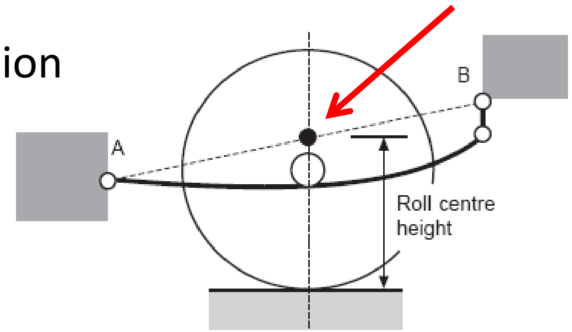


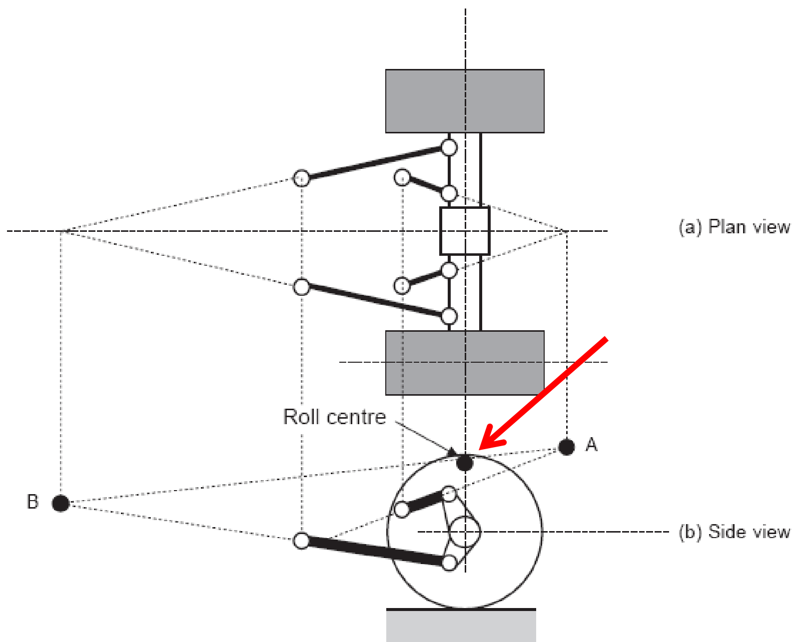
Swing Axle

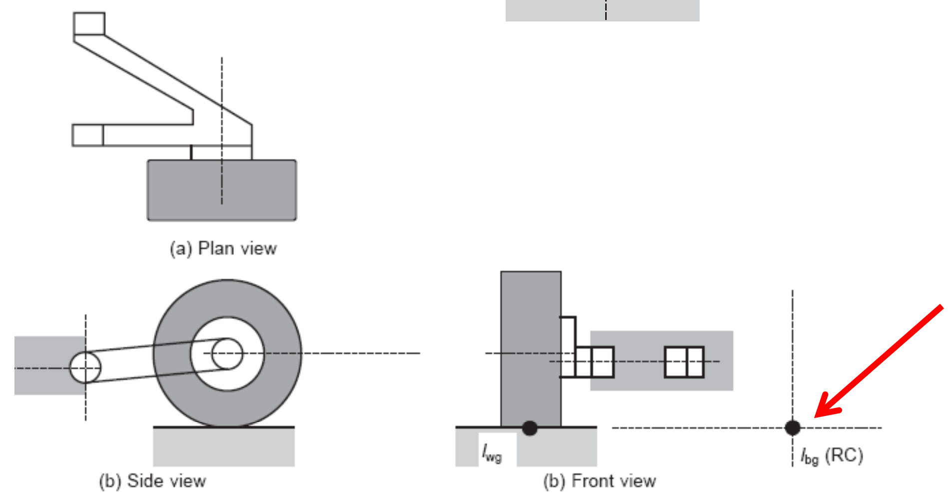
Figure from Smith,2002

Roll-centre determination

Roll centre location for a Hotchkiss suspension







Roll centre for a four link rigid axle suspension

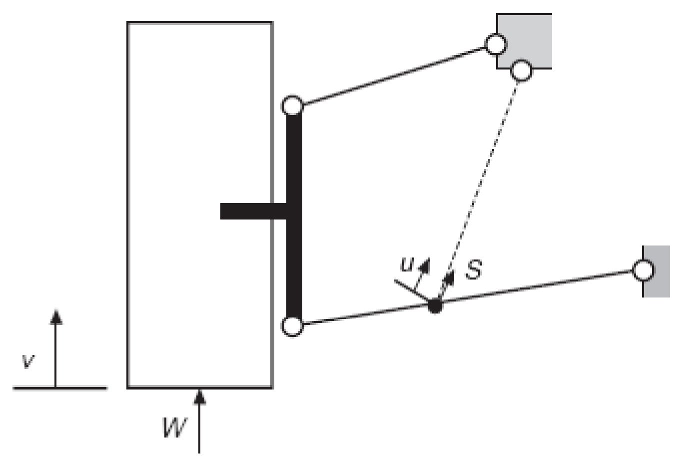
Roll centre location for semi-trailing arm suspension

Figure from Smith,2002

Force Analysis - spring and wheel rates

- Relationship between spring deflections and wheel displacements in suspensions is non-linear
- Desired wheel-rate (related to suspension natural frequency) has to be interpreted into a spring-rate

W and S are the wheel and spring forces respectively
v and u are the corresponding deflections



Notation for analyzing spring and wheel rates in a double wishbone suspension

Spring and wheel rates

Begin by defining the suspension ratio as: $R = \frac{S}{W}$

The spring stiffness is: $k_s = \frac{dS}{du} = d(RW) = R\frac{dW}{dv}\frac{dv}{du} + W\frac{dR}{dv}\frac{dv}{du}$

From principle of virtual work

$$S\,du = W\,dv$$

$$R = \frac{S}{W} = \frac{dv}{du}$$

Wheel rate

$$k_w = \frac{dW}{dv}$$

Spring and wheel rates

Combined Equation is

$$k_s = k_w R^2 + S\frac{dR}{dv}$$

Similarly can be derived for other suspension geometries

Wheel-rate for constant natural frequency with variable payload

Simplest representation of undamped vibration

$$\omega_n = \sqrt{\frac{k_w}{m_s}}$$

k_w – wheel rate

m_s – proportion of un-sprung mass

Change in wheel rate required for change in payload.

Static displacement

$$\delta_s = \frac{m_s g}{k_w}$$

To maintain w_n constant, the static deflection needs to be constant. Combining both equations

$$\frac{W}{dW/dv} = \delta_s = \text{constant, or } \frac{dW}{W} = \frac{dv}{\delta_s}$$

Wheel-rate for constant natural frequency with variable payload

Integrating the equation and substituting with initial conditions provides the following expression

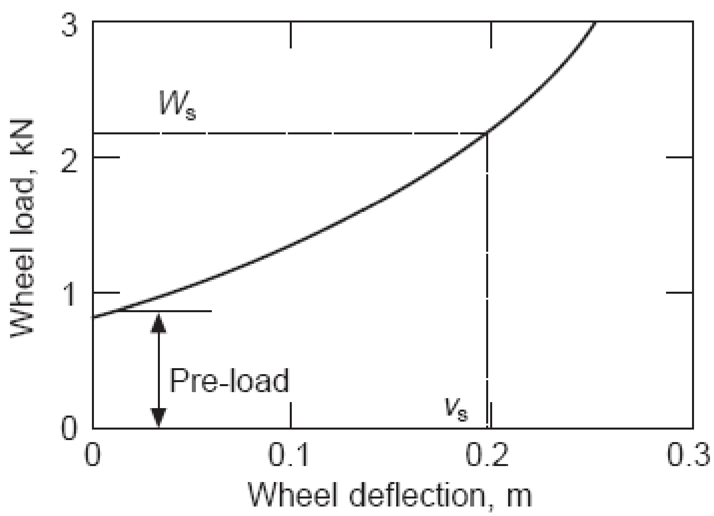
$$W = W_s e^{\frac{v - v_s}{\delta_s}}$$

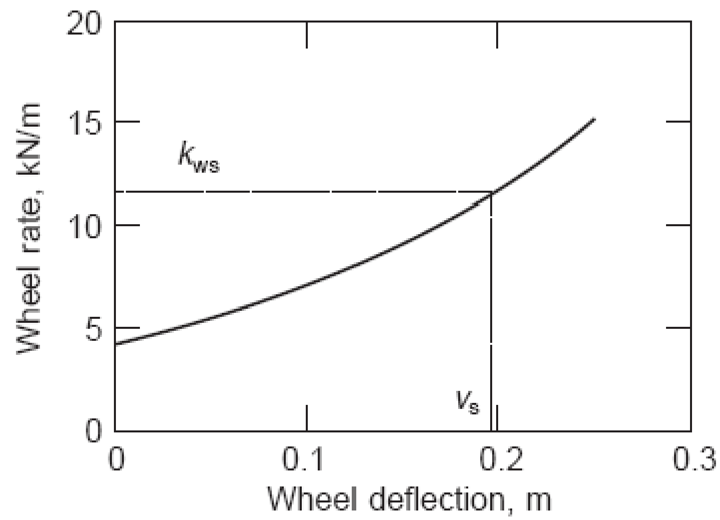
Substituting back , we obtain

$$k_w = \frac{dW}{dv} = \frac{W_s}{\delta_s} e^{\frac{v - v_s}{\delta_s}}$$

Wheel-rate for constant natural frequency with variable payload

Wheel load v. wheel deflection **Wheel rate v. wheel deflection**



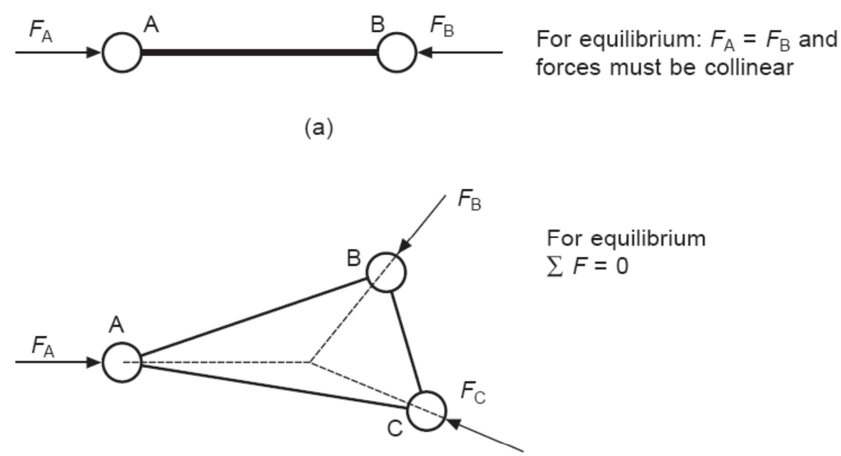


Typical wheel load and wheel rate as functions of wheel displacement

Figure from Smith,2002

Forces in suspension members - Basics

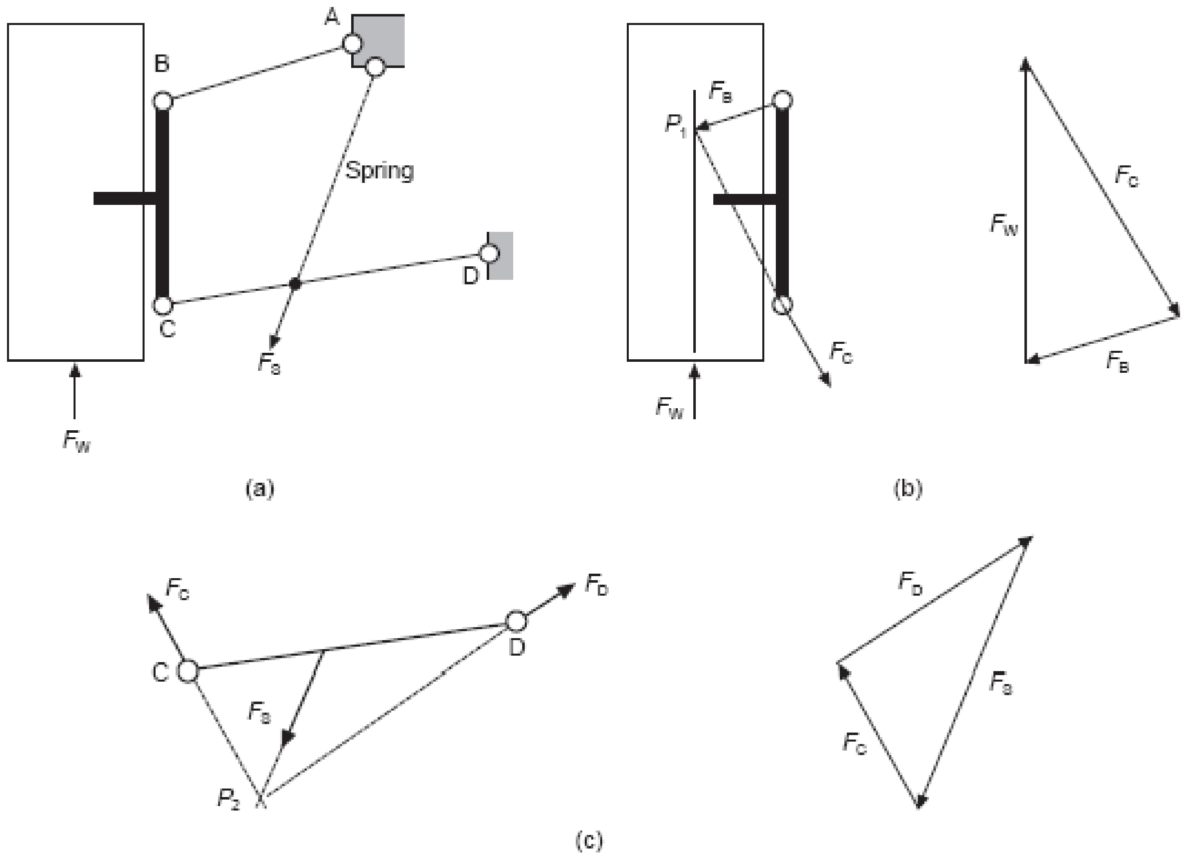
- Mass of the members is negligible compared to that of the applied loading.
- Friction and compliance at the joints assumed negligible and the spring or wheel rate needs to be known
- Familiar with the use of free-body diagrams for determining internal forces in structures
- Conditions for equilibrium



Equilibrium of two and three force members, (a) Requirements for equilibrium of a two force member (b) Requirements for equilibrium of a three-force member

Figure from Smith,2002

Vertical loading



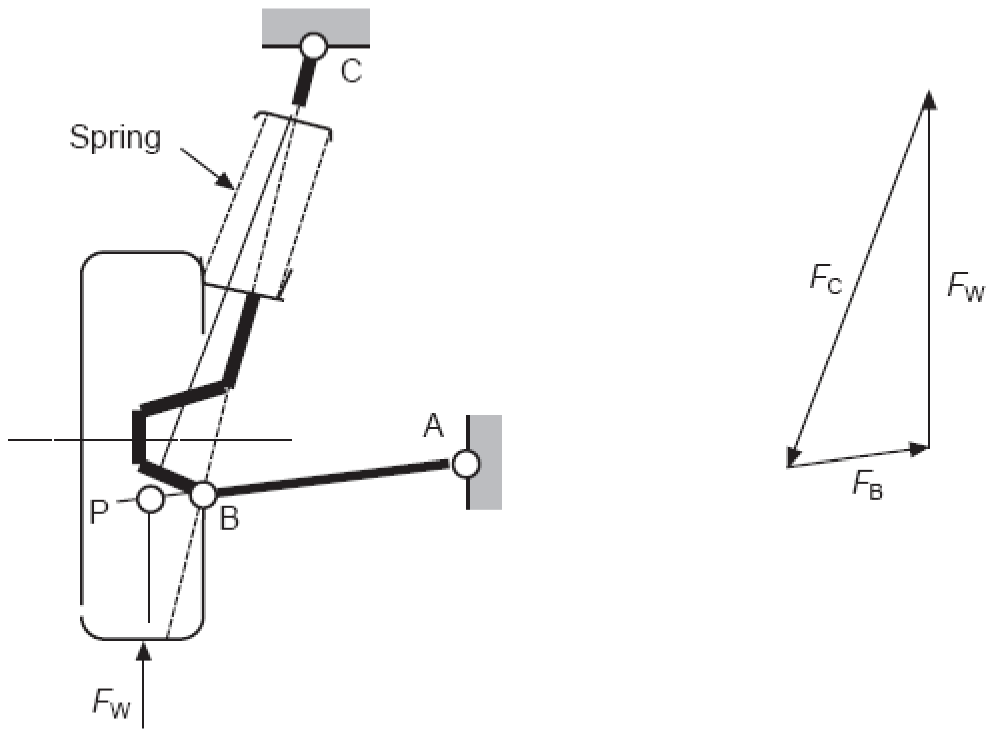
Force analysis of a double wishbone suspension (a) Diagram showing applied forces (b) FBD of wheel and triangle of forces (c) FBD of link CD and triangle of forces

Figure from Smith,2002

Vertical loading

- Assume F_W is the wheel load and F_S the force exerted by the spring on the suspension mechanism
- AB and CD are respectively two-force and three force members
- F_B and F_C can be determined from concurrent forces
- Similar analysis possible for other types also.

Vertical loading- Macpherson



Force analysis of a MacPherson strut, (a) Wheel loading, (b) Forces acting on the strut

Figure from Smith,2002

Forces in suspension members

Lateral loading

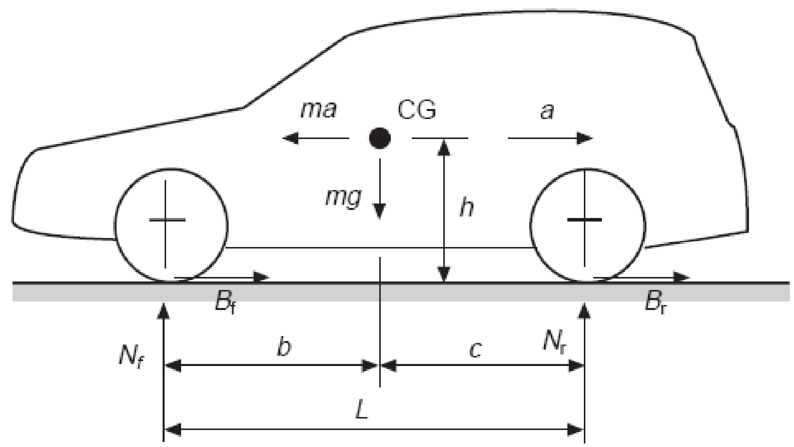
- Lateral loading arises from cornering effects, while longitudinal loadings arise from braking, drag forces on the vehicle and shock loading due to the wheels striking bumps and pot-holes.
- The preceding principles can also be used to analyze suspensions for these loading conditions

Forces in suspension members -Shock loading

Load case	Load factor		
	Longitudinal	Transverse	Vertical
Front/rear pothole bump	3 g, at the wheel affected	0	4 g, at the wheel affected, 1 g at other wheels
Bump during cornering	0	0	3.5 g at wheel affected, 1 g at other wheels
Lateral kerb strike	0	4 g front and rear wheels on side affected	1 g at all wheels
Panic braking	2 g front wheels 0.4 g rear wheels	0	2 g front wheels, 0.8 g rear wheels

Anti Squat / Anti-dive

- During braking there is a tendency for the sprung mass to "dive" (nose down) and
- During acceleration the reverse occurs, with the nose lifting and the rear end "squatting"



Free body diagram of a vehicle during braking

Figure from Smith,2002

Anti-squat / Anti-dive

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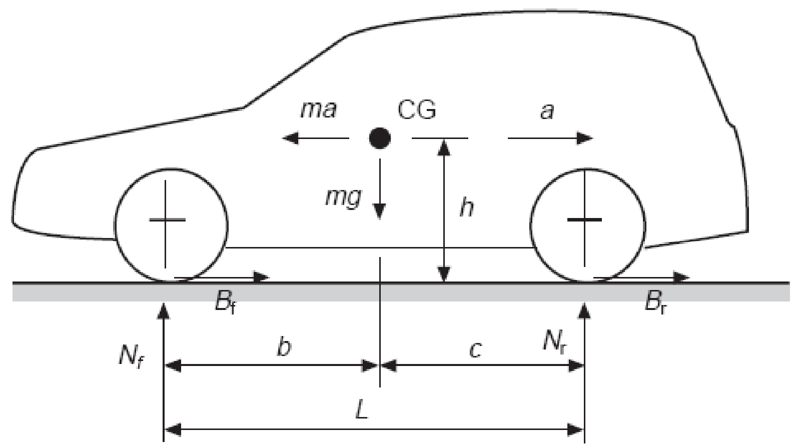


Free body diagram of a vehicle during braking

Figure from Smith,2002

Anti-squat / Anti-dive

D'Alembert force (a pseudo-force sometimes called the *inertia force*) ma, tends to oppose the deceleration. The forces at each pair of wheels comprise normal and braking components.

Assume that there is a fixed braking ratio k, between front and rear braking forces:

$$k = \frac{B_f}{B_f + B_r} \tag{10.14}$$

Under braking conditions the vertical loads on the axles differ from the static values. Take moments about the rear tyre contact point giving:

$$N_f L - mah - mgc = 0$$

Re-arranging gives:

$$N_f = \frac{mgc}{L} + \frac{mah}{L} \tag{10.15}$$

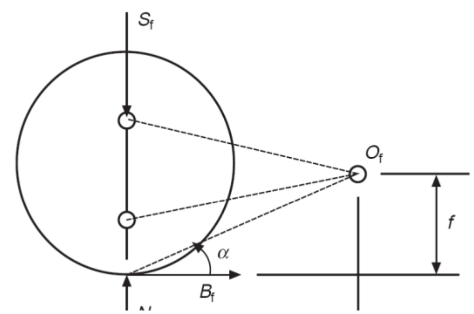
Anti-squat / Anti-dive

The first term on the right hand side is the static load and the second term is the increase in load, i.e. load transfer, due to braking. The corresponding vertical force at the rear is

$$N_r = \frac{mgb}{L} - \frac{mah}{L} \qquad (10.16)$$

The overall effect is an increase in load at the front and a decrease at the rear producing a tendency for dive. Consider now the front suspension with inclined links such that the wheel effectively pivots about O_f in the side view (Figure). The suspension spring force S_f, may be expressed as the static load S_f plus a perturbation dS_f, due to braking, i.e. $S_f = S_f + \delta S_f$ where

$$S_f = \frac{mgc}{L}$$



Under static load conditions ($a = 0$), the spring load is $S_f = \frac{mgc}{L}$. Taking moments about O_f produces: $N_f e - S_f e - B_f f = 0$. Substituting for N_f and S_f and setting $dS_f = 0$ for zero dive gives:

Figure from Smith,2002

Anti-squat / Anti-dive

$$\frac{mahe}{L} - B_f f = 0$$

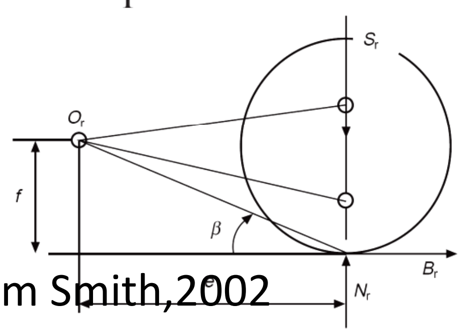
But $B_f = mak$. Substituting this into equation 10.17 and re-arranging gives

$$\frac{f}{e} = \frac{h}{kL} = \tan \alpha \qquad (10.18)$$

If O_f lies anywhere on the line defined by equation 10.18, the condition for zero deflection at the front suspension is satisfied. If O_f lies below this line, i.e. on a line inclined at an angle α' to the horizontal, then the percentage anti-dive is defined as:

$$\left(\frac{\tan \alpha'}{\tan \alpha}\right) \times 100\% \qquad (10.19)$$

A similar analysis for a rear suspension having the geometry shown in Figure leads to an additional equation:



$$\frac{f}{e} = \frac{h}{L(1-k)} = \tan \beta \qquad (10.20)$$

Figure from Smith,2002

Anti-squat / Anti-dive

- If O_r lies on the line defined by equation 10.20 there is no tendency for the rear of the sprung mass to lift during braking.
- It follows that for 100% anti-dive, the effective pivot points for front and rear suspensions must lie on the locus defined by equations 10.18 and 10.20 (shown in Figure)
- If the pivots lie below the locus less than 100% anti-dive will be obtained.
- In practice anti-dive rarely exceeds 50% for the following reasons:
  - Subjectively zero pitch braking is undesirable
  - There needs to be a compromise between full anti-dive and anti-squat conditions
  - Full anti-dive can cause large castor angle changes (because all the braking torque is reacted through the suspension links) resulting in heavy steering during braking.

Anti-squat/ Anti-dive

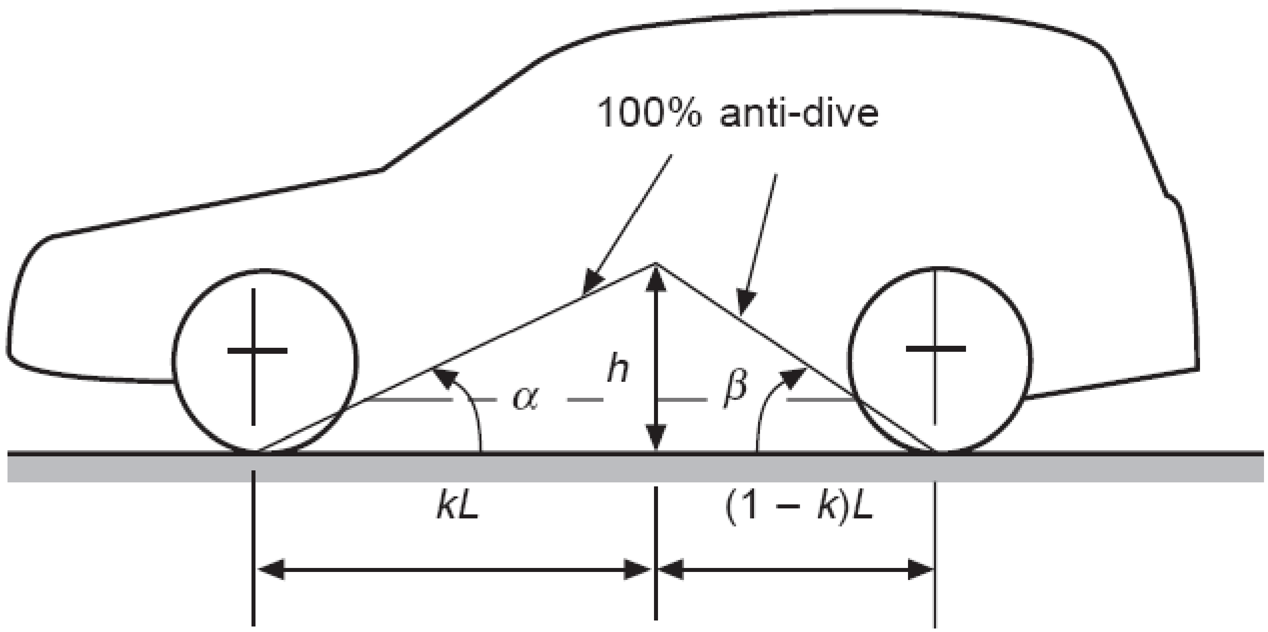


Figure from Smith,2002